***Postage Stamp Problem***



**Setting**: December 2000, the mail room of EightCo, a large company that still uses "snail mail" to mail its product -- styrofoam 8's that are used in packaging.

**Characters:**

**Carla:** The mail room supervisor

**Sandy:** The mail room "gofer"

The scene opens with Sandy returning from one of her many trips to the post office to mail outgoing packages and buy supplies.

**Sandy:** Phew! Sorry it took me so long, but the lines at the post office were terrible.

(Carla frowns)

**Sandy:** I have good news, though. I bought a year's supply of stamps! Since I waste so much time waiting in line to buy stamps, this is going to save me lots of time next year.

**Carla:** How much did you spend?

**Sandy:** Oh, we got those new corporate credit cards and my limit was $6,000, so I bought $3,960 worth of 33-cent stamps and $2,000 worth of 20-cent stamps. That leaves us with $40 for a rainy day.

Carla jumps from her chair and waving her arms in the air.

**Carla:** You knucklehead! Your rainy day is here already. Didn't you hear that the cost of mailing a letter is going to be 34-cents starting next week? Now you'll need to go out and buy some 1-cent stamps. And you'll be the one to lick them all!

**Sandy:** OK, let's see, I spent $3,960 on 33-cent stamps. That's 12,000 stamps, so I'll need to buy that many 1-cent stamps, so I'll need -- 12,000 times 0.01 dollars is... I guess I just move the decimal point on the 12,000 over two places -- $120!

**Carla:** And how much more can we spend on our credit card? I'm not bailing you out of this. You've got to go to the boss and explain what happened.

**Sandy:** Wait! We might be OK after all. Lots of things we mail need much more postage than the basic 34 cents. Like this package [lifts a package out of the bin]. With the new rates, how much will it cost to send this one?

**Carla:** Six dollars and eighty cents. Twenty 34-cent stamps will do it if my dingbat assistant had asked me before spending all our money on 33-cent stamps.

**Sandy:** But wait! If you put 20 33-cent stamps on the package, we would just need to add a 20-cent stamp - we've got plenty of those - so we have the correct postage!

Fade, as Carla is heard mumbling something about hiring a mathematician for the mail room.

Is Sandy right? Will they be able to put the correct postage on most of the packages that EightCo sends out? Specifically:

1. What amounts of postage can they make without buying more stamps?
2. What is the largest amount of postage they cannot make?
3. More generally, suppose the mail room only has stamps of denominations a and b, where a and b are relatively prime integers (i. e., a and b have no common factor greater than 1).
   1. What amounts of postage could the mail room staff make?
   2. What is the largest amount of postage they cannot make?